A Directional Topology Optimisation Method

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1. Abstract

The scope of criterion functions in topology optimisation has so far been limited to performance-related technical aspects, for instance weight, stiffness, or thermal properties. In this work, we extend the use of topology optimisation to the architectural field by consideration of visual expression evaluated by image analysis techniques. The method is based on mathematical programming and allows formulation of problems with any combination of the available structural or visual criterion functions. The focus of this paper is on the choice and implementation of image analysis method for evaluation of the visual criterion.

2. Keywords: Visual expression, architecture, design, multidisciplinary optimisation, image analysis.

3. Introduction

Architectural design of buildings, bridges, and other artefacts is a multidisciplinary activity embracing both technical and esthetical considerations. It is characteristic that what we perceive as good architecture tends to present us with synergistic combinations of technical functionality in many different disciplines and the visual expression of the artefact. In classical architecture, the domes of gothic cathedrals or the elegance of long-span bridges are two well-known cases where the technical requirements and the esthetical qualities unite synergistically. Regrettably, the world is also full of examples of architectural works where synergy is absent, or the wrong balance between the considerations has led to architecturally unsatisfactory results.

The fact that buildings optimised with respect to structural criteria can be visually appealing has been demonstrated [1], and it is foreseeable that the current trend towards new disciplines will eventually enable us to take a wide variety of technical aspects, such as acoustics, natural convection, or flow of people, into consideration in architectural design optimisation. However, to use design optimisation on a wider scale in architecture, the choice of criterion functions must also progress into other, less technical, fields such as aesthetics or visual expression. To bridge this gap between engineering design based on technical criteria and design with respect to esthetical or artistic criteria is the purpose of the present work. By combining the visual expression of the design with engineering aspects we hope to obtain a tool that embraces both the technical and esthetical aspects of products. The work is a project under the Danish Centre for Integrated Design, a co-operation between the Aarhus School of Architecture (AAA), and Aalborg University (AAU), both located in Denmark. To translate the esthetical or artistic criteria into quantifiable properties, we focus our work on analysis of the visual expression of topology-optimised artefacts as it can be evaluated by image analysis techniques. Architectural concepts like "direction", "movement", and "flow of space" play important roles in the practical design of architectural structures that must function in the context of an existing landscape or urban environment. As an initial step, we approach this complex of problems by development of a new criterion related to the object’ s visual direction as defined by its shapes and edges, thus enabling topology optimisation where the visual direction of an object plays an important role.

4. Determination of criteria

The following analysis motivates the choice of visual direction as the first attempt to incorporate visual expression as a criterion in the design optimisation process. It must be clearly understood that the analysis of the visual expression is only a small subset of the criteria handled by architects in the design process. The following figure shows how the analysis of built form can be divided into classes [2]. Each class is analysed for influences on other classes. This way, a hierarchical classification is obtained.

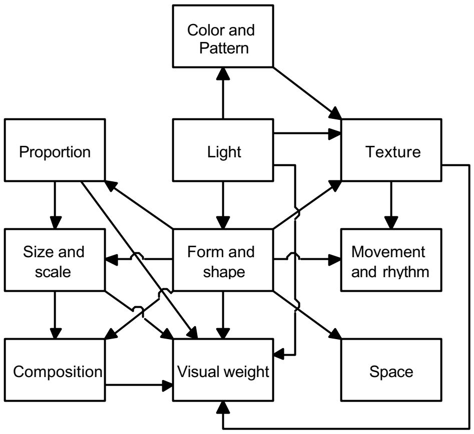


Figure . The flowchart of the interacting classes.

Table 错误！不能识别的开关参数。. High level in the complexity hierarchy means high complexity

Class	In	Out	Hierarchical level
Color and Pattern	1	1	3
Proportion	1	1	3
Light	0	4	1
Texture	3	2	8
Size and scale	2	2	5
Form and shape	1	7	2
Movement and rhythm	2	0	7
Composition	2	1	6
Visual weight	6	0	9
Space	1	0	4

错误！不能识别的开关参数。 visualises the interaction between the 10 classes. Arrows indicate influence coming from other classes. It is now possible to order the classes hierarchically according to their dependency on - and their ability to influence other classes such that a high degree of independence and high influence on other classes leads to a high ranking. Light is the only class that is not influenced in any way by the other classes. However, to minimise complexity it is chosen to work initially in only two dimensions where light does not make sense, seen from an architectural viewpoint. The form and shape class is the next choice. This class interacts with six other classes, indicating that it would be a fine basis if this class were formalised.

5. To find the form

Edge detection in two-dimensional images is a well-developed technique in the field of image analysis. Any, even simple, image processing software has one or more algorithms for this task, and the technology is also used routinely in the manufacturing industry for identification or sorting purposes. Image analysis and edge detection is therefore an obvious origin for a search for methods for analysis of visual direction. We shall develop a tool based on edge detection capable of finding forms and shapes in a topology optimisation model and use it for analysis and sensitivity analysis of visual direction.

Several different algorithms have been tested for reliability. The following four statements characterise a good edge detection algorithm [3]:

1. Good detection. There should be a minimum number of false negatives and false positives.
2. Good localisation. The edge location must be reported as close as possible to the correct position.
3. Only one response to a single edge.
4. Speed. The algorithm should be fast enough to be usable in an optimisation system that uses iterative evaluation of criterion functions.

The algorithm used is based on the idea of the SUSAN (Smallest Univalue Segment Assimilating Nucleus) algorithm [3]. This algorithm is based on a very simple concept different from almost all other edge detecting algorithms [4,5]. Where most ordinary algorithms map between 2 and 8 different masks onto the picture to detect the edges, SUSAN obtains the same goal by just one mapping. In the following, we shall assume that the image is given as a greymap consisting of a regular matrix of pixels, each with an individual greylevel. SUSAN now iteratively applies a mask of some dimension, say 3x3 with centre in each pixel of the greymap. The algorithm then consists of the following steps:

1. Conceive the greylevel/density of each pixel in the mask as mass of a unit area.
2. Find the centre of gravity of the mask.
3. The vector from the centre of the mask to the centre of gravity generates a normal to the edge.
4. The length of this vector gives the response of the analysed pixel.

The vector, $\mathbf{v}_{cg} = (x_{cg}, y_{cg})$, in *xy* space from the geometrical centre to the centre of gravity of each mask is computed as

$$x_{cg} = \frac{\sum_{i=1}^{n} x_i \rho_i \Delta A}{\sum_{i=1}^{n} \rho_i \Delta A} \qquad y_{cg} = \frac{\sum_{i=1}^{n} y_i \rho_i \Delta A}{\sum_{i=1}^{n} \rho_i \Delta A} \tag{1}$$

where n is the number of pixels in the mask, ρ_i is the element density and ΔA may be left out of the equation if we use a regular quadratic greymap. The information supplied by this vector is a direction and a weight, i.e., the length of the vector, for each pixel, where the weight is a measure of the pixel’s quality as an edge. Masks in areas of uniform density will have centres of gravity close to the geometrical centre, and the weight will therefore be small. By statistical processing of the output, in the present case finding the weighted mean value of the direction, it is possible to compute both the deviation from the desired direction and to quantify the length of the edges forming the direction. Consequently, an optimisation problem to minimise the deviation from a desired direction in the picture may be formulated as follows:

$$\min: \; D = \sum_{i=1}^{N} (\theta_0 - \theta_i)\eta_i \tag{2}$$

where N is the number of pixels in the analysed greymap, θ_0 is the desired visual direction, θ_i is the calculated direction and η_i is the weight factor reflecting the length of each pixel’s CG vector. However, to avoid instability in the algorithm due to the periodic nature of trigonometric functions, it is advantageous to formulate the problem in terms of vector algebra instead of angles.

$$\min: \; D = \sum_{i=1}^{N} \left(\begin{pmatrix} \cos\theta_0 \\ \sin\theta_0 \end{pmatrix} \times \begin{pmatrix} x_{cg} \\ y_{cg} \end{pmatrix}_i \right) \tag{3}$$

6. Regularisation

The image analysis procedure described above is only applicable to a regular matrix of pixels. If the greymap is in fact a finite element model with varying densities in each element, this would translate into a mesh of quadratic elements. In the context of a versatile design optimisation tool it is desirable to be able to handle problems with irregular, non-quadratic meshes. To this end, a translation between the densities of a possibly irregular finite element mesh and the greymap has been devised. This regularisation is produced by use of a modified version of the cumulative approximation scheme [6]. This approximation produces an explicit expression for the interpolated density $\rho(\mathbf{p})$ at any point $\mathbf{p} = (x,y)$ of the design domain:

$$\rho(\mathbf{p}) = \frac{\sum_{k=1}^{K} \Phi^{(k)}(\mathbf{p})\rho_k}{\sum_{k=1}^{K} \Phi^{(k)}(\mathbf{p})} \tag{4}$$

where ρ_k is the density of the k’th of totally K finite elements in the model. The $\Phi^{(k)}$ functions blend the influences of all elements originating in their centroids:

$$\Phi^{(k)}(\mathbf{p}) = \exp\left[\frac{-d^{(k)}(\mathbf{p})}{s^2} \right] \tag{5}$$

where s is a characteristic size of the problem and $d^{(k)}$ is the square distance from the centroid, $(x^{(k)},y^{(k)})$, of the k’th finite element to $\mathbf{p}$:

$$d^{(k)} = \left(x - x^{(k)}\right)^2 + \left(y - y^{(k)}\right)^2 \tag{6}$$

The nature of the blending functions $\Phi^{(k)}$ is such that their influence vanishes as the distance from $(x^{(k)},y^{(k)})$ increases. This means that the density of a given finite element reigns in the neighbourhood of that element’s centroid, whereas neighbouring elements gradually take over as their centroids are approached. The blending between elements can be controlled by means of the characteristic distance s. Small values of s will cause the interpolated density to change rapidly between the elements, and larger values of s will smooth the transitions between elements as shown in figures 1 and 2.

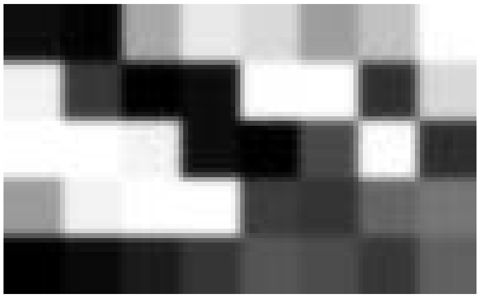


Figure Greymap interpolated with a small s value.

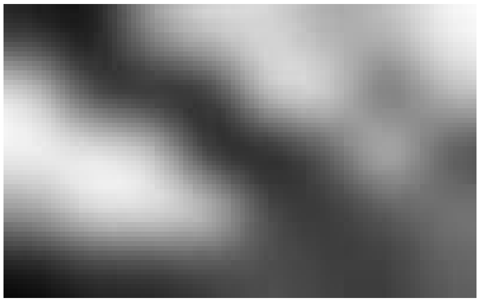


Figure Greymap interpolated with a larger s value.

The regular greymap for the image analysis can now be obtained by imposing a regular grid on top of the finite element model and compute the interpolated grey value at each grid point. Please notice that the interpolation is explicit and therefore computes relatively quickly.

7. Efficiency

The computed visual properties are used together with structural properties such as compliance and volume in a topology optimisation framework. When working with multiple and varying combinations of criteria, it is advantageous to solve the topology optimisation problem by means of mathematical programming rather than optimality criterion methods. This implies the use of sensitivity analysis with respect to the design variables, i.e., the material density of each element in the structure. To obtain these sensitivities by finite differences with repeated calls to the image analysis function would lead to prohibitive computation times. To avoid this problem, data structures are set up to link each design variable with the subset of the image in which the density influences the edge detection. This region is guided by the size of the characteristic distance s in (5). This enables local recalculation of the picture in the sensitivity analysis and reduces the total cost of all sensitivities to the order of K rather than K^2, where K is the number of design variables, i.e., finite elements.

There are two time-expensive phases in the algorithm, these are:

1. The repeated evaluations of the exponential function in (5).
2. The updating of the entire regular greymap each time the density of a finite element is perturbed.

To minimise the use of the computationally expensive exponential function a self-mounting updating scheme has been constructed and implemented. The scheme performs one full re-evaluation of all greymap pixels for all element density perturbations in the first round of sensitivity analyses. The response of each pixel with respect to each design variable is recorded, and in subsequent sensitivity analyses, only those pixels with responses above a given threshold are recomputed. Please notice that the blending functions depend only on the spatial co-ordinates, and the identification of "active" pixels to each design variable is therefore valid throughout the process regardless of the changes of density distribution.

The advantages of using a self-mounting updating scheme is that all affected pixels are updated, no more no less. The disadvantage is that no gain of the updating scheme is to be seen until the second iteration of the optimisation process. In the first iteration the entire greymap is calculated K+1 times, where K is the number of design variables.

8. Results

To illustrate the method, an academic and a more practical example have been developed. The Academic example concerns the classical topology design case of an 8 x 5 cantilever beam with a vertical point load in the middle of its extreme edge. Figure 3 shows the well-known solution to the problem when the compliance is minimised with a bound on the volume and a perimeter constraint. Figure 4 shows the same ground structure subjected to an optimisation of visual direction to approach 45 degrees with the same volume fraction and a bound on the compliance of two times the optimised value of Figure 3. This problem has no perimeter constraint.

The result shows clearly that the optimisation process has produced a compromise between stiffness and visual direction with an emphasis on the latter dictated by the relaxation of the compliance requirement. The algorithm even positions superfluous material with no structural function just to improve the amount of 45 degree edges. It appears that, to obtain the desired synergistic relationship between structure and visual direction, the right compromise between the two criteria must be specified in the optimisation problem formulation.

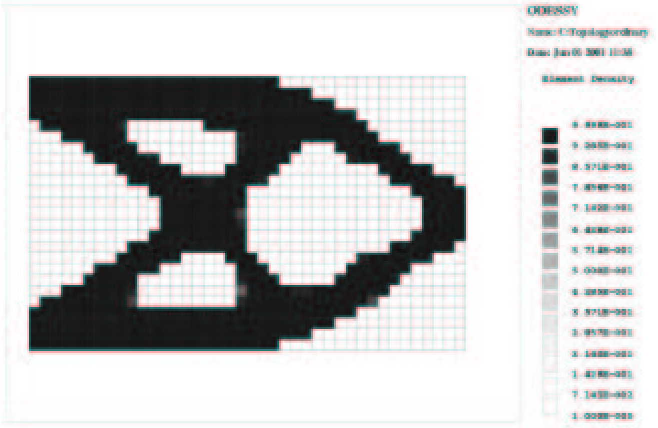
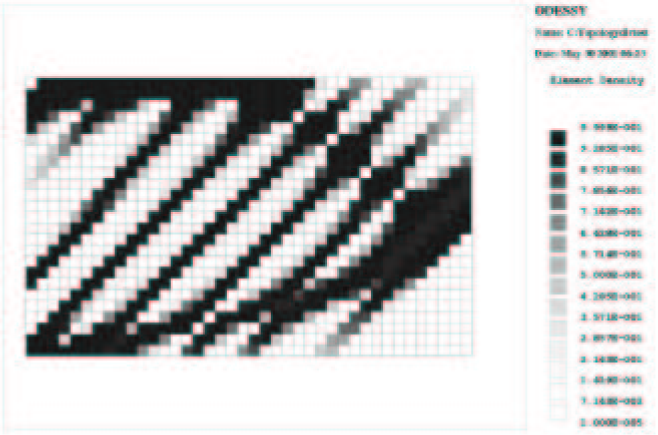


Figure Ordinary compliance optimisation problem.



Figur An 45 degree optimised cantilever beam, with constraint on volume fraction and compliance.

Visual direction plays an important role in the design of several structures and products. Sports cars and ships have "lines" that indicate their velocity in a given direction, and a bicycle frame with as few members as possible breaking the wind and creating unnecessary drag is very desirable. Another typical example is the blending of buildings and bridges into an existing landscape. Figure 6 shows a bridge with a landscape slope behind it. The design of this bridge can be obtained by ordinary topology optimisation as illustrated in Figure 7 when the surrounding landscape is not taken into account. Figure 8 shows the landscape with the bridge removed and the landscape' s slope of approximately 5 degrees. Optimising the visual direction of the bridge to approach 5 degrees while honouring a compliance constraint of 150% of the original bridge produces the result of Figure 9.

As with the first example, the algorithm produces the desired result in terms of direction, but the synergy between direction and structure is not obvious. Furthermore, some amount of checkerboard formation is visible.

Figure Old bridge over the bay.

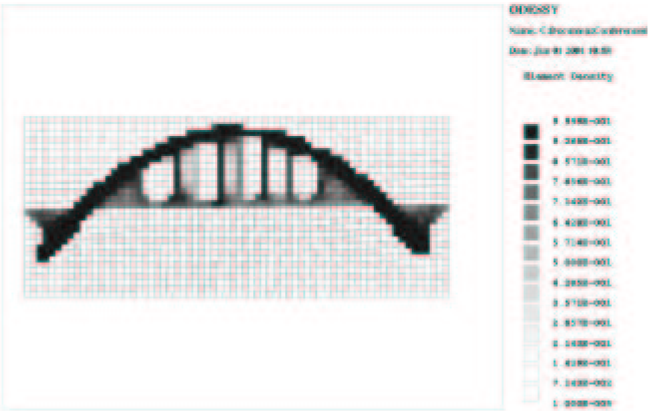


Figure Ordinary compliance optimisation problem.

Figure Retouched bay with the wanted direction indicated by the black line.

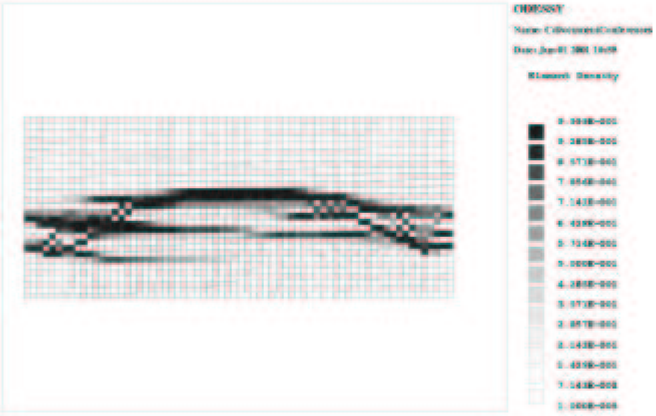


Figure Bridge optimised for 5-degree directions. Objective function: Minimum direction difference. Inequality constraints on volume of material and compliance.

9. Conclusion

A direction criterion was implemented into an existing topology optimisation code ODESSY [7]. The method works, but the results are not yet convincing in terms of the esthetical and synergistic qualities that are the true goals of the work. On the other hand, the development is still at an early stage, and further experiments with problem formulation, finer meshes, perimeter constrains, etc., are expected to improve the results considerably.

10. Acknowledgements

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11. References

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